

Applications of Machine Learning to One-Dimensional Problems of Mechanics

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Abstract:

A new trend in machine learning research is neural networks incorporating physical governing equations as constraints. In this vein, we provide a neural network constraint based on the governing equation for a deep learning model for one-dimensional consolidation. Prior research is reviewed and discussed first. Automatic differentiation is used by the deep learning model to restrict the application of the governing equation. Analytical and model-predicted solutions, as well as constraints, are used to calculate total loss (a requirement to satisfy the governing equation). Forward and inverse difficulties are both taken into account. For one-dimensional consolidation issues, the forward tasks show how well a neural network model with physical constraints performs in prediction. The coefficient of consolidation may be predicted using inverse problems. As an example, we employ Terzaghi's problem with shifting boundary conditions, and the deep learning model displays a fantastic performance in both the forward- and inverse-problem scenarios. A deep learning model with physical law integration may be useful for a wide range of applications, including faster real-time numerical prediction for digital twins, reproducibility of numerical models, and optimization of constitutive model parameters, although this particular application is a simple one-dimensional consolidation problem.

1. Introduction

Even outside of the realm of pure computer science, the application possibilities for machine learning have expanded rapidly in recent years. Artificial neural networks, a specific form of deep learning, are being used successfully in a variety of fields of science and industry. Recent years have seen a rise in the use of deep learning in the context of partial differential equations (PDEs). Several researchers are involved in this endeavour, and the use of deep learning in conjunction with physical systems regulated by PDEs is referred to by a variety of names. Physics-informed neural networks, theory-guided data science, and deep hidden physics models are just a few of the prevalent designations. Numerical techniques for solving partial differential equations (PDEs) may be more efficient, accurate, and generalizable with the help of these applications.

Deep learning and machine learning may be used to solve one-dimensional consolidation difficulties. Fluid movement and excess pore water pressure dissipation in porous media are described in the issue. The problem's governing equation is briefly explained. The governing partial differential equation's deep learning model is then explained. For both forward and inverse tasks, outcomes are reported.

Consolidation theory explains how compressive stress delays porous media deformation by dissipating fluid from the porous medium. One-dimensional consolidation is governed by the equation

$$\frac{\partial p}{\partial t} - \frac{\alpha m_v}{S + \alpha^2 m_v} \frac{\partial \sigma_{zz}}{\partial t} - c_v \frac{\partial^2 p}{\partial z^2} = 0 \quad (1)$$

The vertical effective stress (zz) is equal to the vertical effective pressure (p) plus Biot's coefficient (α), and the pore space storage (S) is equal to the restricted compressibility (m_v) of the porous medium. In the conventional one-dimensional consolidation issue, a compressive force is applied at time $t=0$ and the load is sustained for time $t>0$. A continuous tension zz is seen for time intervals longer than zero. It's therefore possible to simplify the general formula (as in (1))

$$\begin{aligned} \text{For } t = 0 : \quad p &= p_o = \frac{\alpha m_v}{S + \alpha^2 m_v} q \\ \text{For } t > 0 : \quad \frac{\partial p}{\partial t} - c_v \frac{\partial^2 p}{\partial z^2} &= 0 \end{aligned} \quad (2)$$

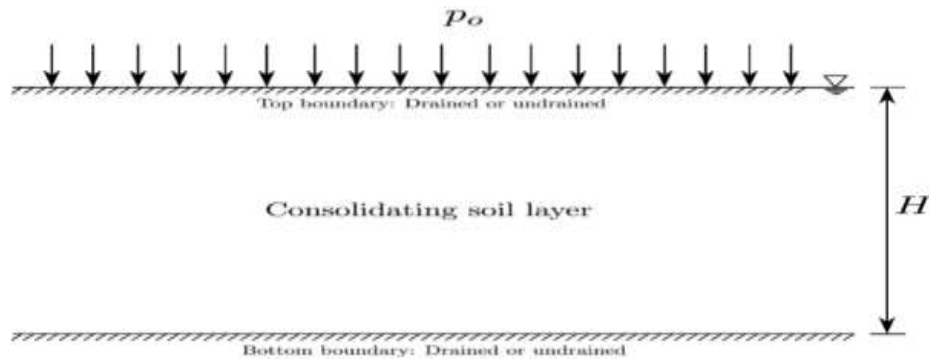


Fig. 1. One-dimensional consolidation.

$t=0$ is the starting point for the 1D consolidation issue, when the complete vertical load is borne by the pore fluid and there is no dissipation from the porous medium. The dissipation rate of the pore fluid is controlled by the second equation, which takes into account time as well as spatial dimension. Analytical or numerical approaches may be used to solve this equation for varying drainage boundary conditions at the top and bottom of the porous material. Analytical solutions are shown here for two alternative drainage boundary conditions, which will be discussed in more depth in a subsequent section.

2. Literature Survey

There is a shift away from conventional statistical approaches and mechanistic models toward ML and DL, such as deep neural networks (DNNs), across many commercial applications and sectors, such as education, natural science, medical research, engineering, as well as social science. mechanical engineering has typically relied on mechanistic models [1]. A common criticism of ML approaches is that they seem to be "black boxes," meaning they take in inputs and produce outputs but do not disclose information that can be understood by the user[2,3]. Black-box models have been widely criticised for their opacity, and some scientists have constructed physics-based ML in response. Even academics who construct the algorithms that produce the mechanical engineering ML models do not comprehend how variables are integrated to make predictions in these models. No matter how many input variables are included in a black-box predictive ML model, no researcher can grasp how the variables are linked together to arrive at the final prediction. High-data-demanding ML models, for example, have trouble estimating structural damage since they are related to processes that aren't fully understood. Thus, their large data requirements, difficulty in delivering physically consistent results, and inability to generalise to out-of-sample events [4,5]. ML and DL model[6]s are tested on large, curated datasets with well-defined, accurately labelled categories. These issues can be handled successfully by DL since it presume that the world is steady. While these classifications are continually developing in the actual world, particularly in mechanical engineering, We can only detect the issue after doing extensive testing on ML responses to diverse visual cues. mechanical engineering applications such as earthquake risk reduction, irrigation management, structural design and analysis, and structural health monitoring need physics-based numerical models. High-performance computers have enabled mechanical engineers and scientists to run ultra-realistic simulations with millions of degrees of freedom in their models for use in the real world. Simulating in the mechanical engineering industry is too time-consuming for an iterative design approach to integrate. Although they are often used during testing and certification, they are commonly used only in the last phases of development. Because numerical tools may be used throughout the design process if they can be accelerated, this is an essential challenge to solve [7,8]. Model complexity has hindered the development of innovative applications, such as enhancing construction productivity, that might benefit from the development of numerical techniques for quick simulations. An further key example of analysis that may be possible if simulation expenses were much reduced is uncertainty quantification. In fact, the variables of interest tracked by numerical simulations are affected by the actual system environment, which is often unknown. It may be necessary to estimate probability distributions for the quantities of interest to ensure the dependability of the product if these uncertainties have a substantial influence on simulation outcomes. For complex scientific and technological applications, neither an ML-only nor a scientific knowledge-only approach may suffice. To better understand the continuum between mechanistic and machine learning models, researchers are trying to combine scientific knowledge with data.

ML mechanical engineering has been the subject of several evaluations. In mechanical engineering, however, only a few studies have focused on the correct use of physics-based machine learning in mechanical engineering and creating a road map for future studies. Few studies have focused on the basic physics-based ML modelling in mechanical engineering. Mathematical modelling (ML) approaches and physics models are examined in depth in this work. Many studies have previously been done on merging scientific ideas with ML models, despite the fact that the idea of doing so has just lately gained traction[9,10]. In mechanical engineering, researchers use physics models, machine learning models, and use cases to tackle their difficulties. This research intends to inform the ML community about these exciting achievements, as well as the gaps and potential for furthering research in this promising path.

3. Deep Learning Model

A physical restriction based on the governing one-dimensional consolidation equation is addressed in this part, along with the neural network design. Hyper-parameters that are controlled during training are also described, as is the model training technique.

3.1 Neural Network Architecture

Using a fully-connected deep neural network with the necessary number of layers and hidden units, a one-dimensional consolidation issue is trained. Neural networks are shown in the diagram displayed in the following image. This one-dimensional consolidation issue uses (z,t) values from the training data as inputs for its input layer; the specifics are covered in the following sections on numerical examples for forward and inverse learning.

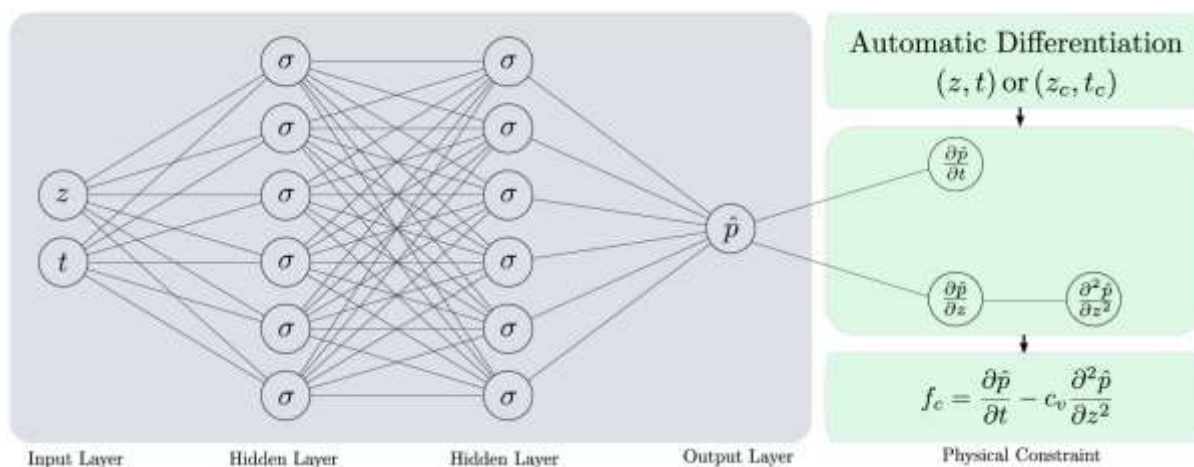


Fig. 2. Illustration of the neural network architecture

Using the excess pore pressure training data, the neural network with the required number of hidden layers and hidden units predicts the excess pore pressure[11]. Also included is a physical constraint, based on the governing one-dimensional consolidation equation, which is assessed via automated differentiation, which is briefly addressed in a subsequent sub-section below. An artificial neural network is built to minimise the training loss while still accommodating a physical limitation.

3.2 Automatic Differentiation

Automatic differentiation is a critical component of the deep learning model used to solve this issue. Automated differentiation should not be confused with other techniques for calculating derivatives in computer systems. Derivatives may be computed in four ways: manually, using finite difference approximations, numerically, using computers to approximate derivatives, and automatically, using computers to calculate symbolic derivatives and then evaluate them using algebraic expressions. Automatic differentiation, like the other approaches, gives numerical values of derivatives where these are derived using the principles of symbolic differentiation, but instead of producing the final expressions, they maintain track of the derivative values. Using this way of monitoring derivative values, automated differentiation is superior to the two most prevalent methods of calculating derivatives, numerical and symbolic differentiation[12,13]. There are just a few simple arithmetic operations and elementary function evaluations involved in each derivative calculation, no matter how sophisticated. The chain rule is continually used until the required derivative is calculated. Using this methodology, automated differentiation may be performed at machine accuracy and is far less computationally intensive than previous approaches. TensorFlow's automated differentiation functionality is used to evaluate the derivatives in the controlling one-dimensional equation[14]. By recording all operations and calculating the gradients of the recorded calculations using reverse mode differentiation, TensorFlow offers an API for automatically differentiating models.

4. Drained Top Boundary

A one-dimensional model with a drained top border, a model height of 1 m and a coefficient of consolidation of $c_v=0.6$ m²/yr is studied here in an inverted situation with the same geometry and material/model characteristics. Using $N_z=100$ and $N_t=100$, the analytical solution is produced again. This means that the precise solution has a total of 10000 points in z , t , and p . The architecture of the neural network is designed to include 10 hidden layers, each having 20 hidden units. The neural network is trained using a random selection of 2000 points from the analytical solution data (which has a total of 10000 points). A batch size of 200 is used to shuffle and split the training data[15]. Initialization of $w_{cv}=0$, the trainable weight corresponding to the consolidation coefficient, implies an initial consolidation coefficient of 1.0 m²/yr. Minimize training and constraint losses by training with a learning rate of 0.0001 and optimising using Adam.

Using the inverse analysis, the outcomes of the issue with the drained upper border are shown in the graph below. As you can see in the top colour plot, white dots represent the randomly picked training data points. The deep learning model predicts the extra pore pressure well from a little training sample data, much as in the forward problem. Another example of the outstanding performance of the physical constraint produced by automated differentiation is provided here as well.

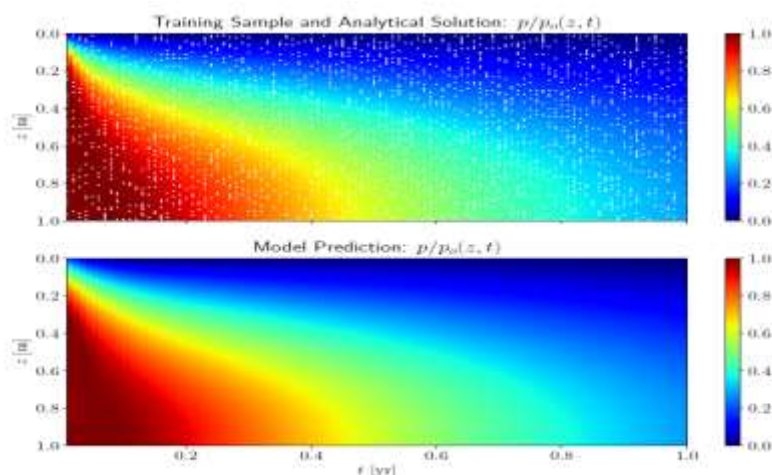


Fig. 3. Inverse analysis results from analytical solution

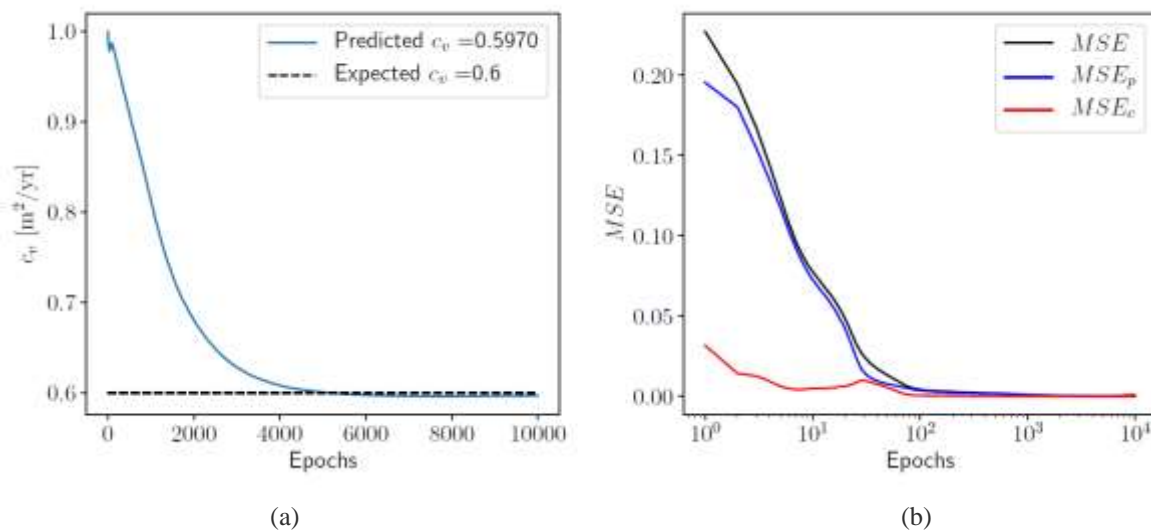


Fig. 4. There are two plots here: (a) a plot showing the projected consolidating coefficient with time and (b) one of the mean squared error vs the number of training epochs.

In the picture above, the projected coefficient of consolidation is shown as a function of training epochs in the plot on the left. $C_v = 0.5970$ m²/yr, which is quite near to the anticipated value of 0.6 m²/yr with an absolute error of $3.010 \cdot 10^{-3}$, is the final projected value for the coefficient of consolidation. On the right, you can see how the mean squared errors have changed over time.

5. Drained Top and Bottom Boundaries

In the last part, we looked at a numerical example in which the top and bottom bounds were both drained. Using a coefficient of consolidation of 0.1 m²/yr, which we hope to anticipate using the inverse deep learning model, we are able to derive the analytical solution for this model. Analytical solution based on $N_z=100$ and $N_t=100$ yields 10000 data points, from which a training sample of 2000 is picked. The hidden layers and 20 hidden units in each layer of the neural network have a similar structure. The batch size is 200 and the Adam optimizer is used with a learning rate of 0.0001 for the other model hyper-parameters.

The figures below illustrate the inverse analysis findings for drained top and bottom bounds. The model accurately forecasts the grid's excess pore pressures even with a little training sample.

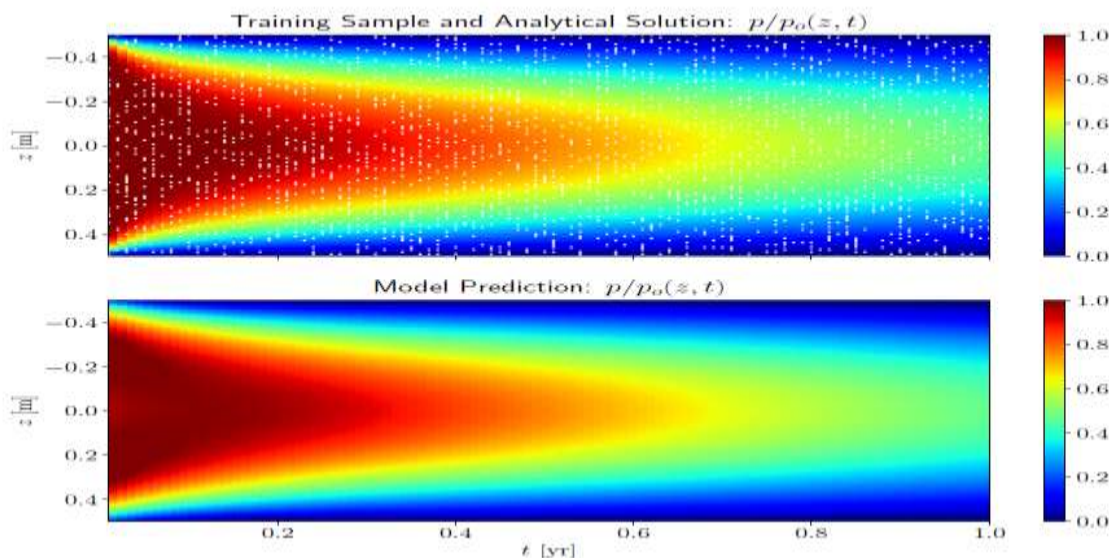


Fig. 5. Inverse analysis results from analytical solution

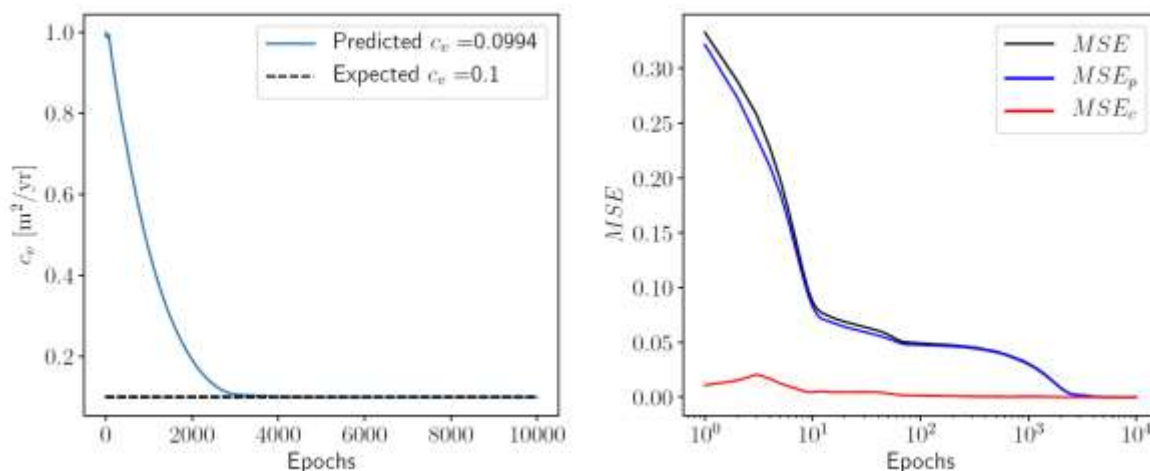


Fig.6. (a) Evolution of the anticipated coefficient of consolidation for drained top and bottom bounds as a function of training epochs; (b) Mean squared error as a function of training epochs.

According to the deep learning model, $c_v=0.0994$ m²/yr, which is an absolute inaccuracy of $6.010 \cdot 10^{-4}$, is the projected consolidation coefficient. These findings may be seen on the left plot of the image above, which depicts the expected coefficient of consolidation over time as a function of training epochs. To get an idea of how many epochs it takes to train a neural network, we plotted the mean squared errors (training and constraint) as a function of epochs.

6. Conclusions

The governing partial differential equation is utilised as a constraint in a deep learning model for one-dimensional consolidation. Researchers in the machine learning field have lately begun to investigate physics-constrained neural networks, and the work

described here contributes to that endeavour. Briefly, a few examples of how the concept has been used in various scientific and technical fields were discussed.

A fully-connected sequential neural network is the deep learning model used here. The extra pore pressure is a function of the geographical and temporal coordinates, which are sent into the neural network. It is possible to confine the system physically by utilising the governing equation of one-dimensional consolidation since the neural network design incorporates automatic differentiation. Using TensorFlow's automated differentiation capabilities, the projected extra pore pressure is assessed in terms of its spatial and temporal derivatives. Forward and inverse difficulties are both taken into account. Derivatives are assessed at a certain number of points created using the latin hypercube sampling approach, taking into account the spatial and temporal boundaries of the real model. A surprising degree of precision might be achieved by training a neural network utilising just the starting and boundary condition data. Training the neural network is done using a larger set of randomly chosen training data points, and the derivatives based on the governing equation are assessed at these points. For solving forward problems, the coefficient of consolidation is used immediately, but it is left as a training parameter when solving inverse ones. The total training loss is defined as the sum of the training and constraint losses for both forward and inverse issues. Excess pore pressure predicted by the model and its accurate analytical solution are used to calculate the training loss's mean squared error. Based on the derivatives and automated differentiation, the mean squared error for constraint loss is calculated using the governing partial differential equation. Minimizing total mean squared error (TMSE) is accomplished by the use of a model optimizer (Adam). The number of hidden layers, the number of hidden units at each layer, the batch size for training, and the optimizer learning rate are all model hyper-parameters that may be changed for any kind of issue. In order to show the model's potential, Terzaghi's one-dimensional consolidation issue with two versions is used: one with simply a drained top boundary and one with drained top and bottom borders. It was shown that the pore pressure across the model's spatial and temporal boundaries could be accurately predicted for forward issues, while the coefficient of consolidation could be accurately predicted for inverse problems using the model.

There are significantly wider ramifications to our deep learning model than can be seen in this basic one-dimensional consolidation issue. Because of the physical constraints, deep learning models trained with a little quantity of data but with high accuracy may predict numerical solutions quicker than models trained with a large amount of data but with low precision. For digital twins, where real-time numerical prediction is desired, this might be a tremendous benefit. Reproducibility and optimization of complicated numerical models might benefit from the ability to forecast material properties, as illustrated by inverse issues.

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