Behavioral theory as a new paradigm for portfolio optimization: Determining the set of possible portfolios and standard portfolio optimization in the presence of ambiguity aversion.

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Summary

One of the most important concerns of investors is to choose the best possible investment opportunity to maximize the value of their investments. However, as shown by Markowi (1952), in an uncertain and risky environment, the decision about the optimal portfolio composition is a complicated process. Risk, ambiguity and uncertainty are sometimes substitutable terms, but their meaning is not always easily understood. Knight (1921) proposed in his treatise "Risk, Uncertainty and Profit" a fundamental distinction between risk (the probability of an event occurring is known) and uncertainty (the probability of an event occurring is unknown). Decision making under risk and uncertainty has been and continues tobe a very important subject that has been the subject of numerous studies, both empirical (actual behavior of individuals) and theoretical (axiomatic of decision theory), in order to develop models of rational decision making and/or taking into account the actual attitude of individuals towards risk. Portfolio optimization is a very important decision for investors.

Key words : Behavioral theory, behavioral biases, portfolio optimization

Introduction

Traditional optimal portfolio selection is based on the principle of investor rationality and risk aversion, which is the basis of classical decision-making theory, based on the fundamental theory of expected utility (von Neumann and Morgenstern, 1944). The latter was developed on the basis of Bernoulli's utility theory (1738), which corresponds to an attitude ofaversion to risk (concave utility function) and has been the subject of innumerable research studies, particularly in finance and in particular in the context of financial asset allocation. Indeed, the expected utility theory is based on the hypothesis of a "homo-economist", whose behavior is governed by self-interest and rational decision-making. It assumes that each decision-maker has the information and resources necessary to find the solution that will bethe best (in some sense to be specified). However, despite its initial popularity among theorists, this theory does not always reflect the reality of investor behavior with respect to therandom fluctuations of financial markets. Over the past few decades, views on these actual behaviors have undergone a shift. Behavioral experiments in laboratories have become an important component of economic research and experimental results have shown that the fundamental assumptions of classical economic theory need to be modified. Indeed, with the evolution of financial markets over the years, the importance of risk has become more and more prominent, especially after the "subprime" financial crisis in 2008, following which global financial markets were confronted with a succession of rapid and violent financial shocks. Thus, the globalization of financial markets, financial integration, improved technology in trading systems and exchange systems have pushed markets to become much more complex while bringing new sources of risk. The growth of market globalization has made the environment more volatile, leading to companies, financial institutions and investorsbeing exposed to more uncertainty in addition to traditional financial risks (see in particular the problems of risk contagion). These negative consequences that the financial markets have experienced, following the succession of crises and financial shocks, have changed the behavior of investors and have called into question the theory of classical finance, based on the hypothesis of the rationality of the investor's choice, and in particular the so-called "independence axiom".

This change gave rise to a new stream of research in the world of economics in the early 2000s, called "behavioral economics", which takes into account the importance of emotions and feelings in individual choices. Behavioral economics, which is considered a combination of psychology and economics, is based in particular on the theory of perspectives(Kahneman and Tversky, 1979). In terms of application, it leads naturally to behavioral finance. Research in behavioral finance has gained momentum since the 2000s, as traditional finance researchers were unable to explain several empirical phenomena, such as the financialbubbles in Japan, Taiwan, the United States, etc.

It is seen as a new approach that has emerged in response to the difficulties encountered by the traditional expected utility paradigm in financial markets. It is presented as a component of behavioral economics and allows for a better explanation of real events than classical finance. Indeed, it explores the behavior of individuals, directly or indirectly, byexamining the different attitudes of agents, their desires, their errors, their preferences and their behavior through experimental tests. Thus, it is defined as the study of how psychology influences the behavior of individuals, at the individual or collective level, in financial markets (Sewell, 2010). As for Statman (2014), he defined behavioral finance as a paradigm that incorporates some parts of classical finance and substitutes some others. It provides a bridge between theory and practice, while embracing the scientific rigor introduced by standard finance. Statman (2014) found, among other things, that financial choices are influenced by the culture, social responsibilities, and feelings of each agent. He also points outthat behavioral finance is being built as a strong structure in the field of finance. This relatively new stream of research has been driven by cognitive psychologists who have studied individuals' judgments in decision-making and by experimental economists who have tested models of economic decision-making. They showed that cognitive biases (i.e. mental accounting, loss aversion, overconfidence...) and heuristics are very important in the decision-making process. This new science is considered a critique of the paradigm of classical financewhich is essentially based on the rationality of the individual's choice and the efficiency of information at the price level.

In 2002, the Nobel Prize in Economics was awarded jointly to economist Vernon Smith and cognitive psychologist Daniel Kahneman. Kahneman is considered the pioneer of behavioral economics, and in particular of behavioral finance, for his research contributions inempirical psychology to the science of economics, and in particular to the field of decision making. Indeed, he addressed the cognitive and emotional biases that generate the anomalies that occur in the stock market and developed the prospect theory that is considered one of the foundations of behavioral finance. Indeed, the theory of perspectives was developed by Kahneman and Tversky in 1979. It was later refined by Kahneman and Tversky (1992) to become the cumulative prospect theory (CPT). This theory has become a standard model based, among other things, on the probability weighting function. It allows for several experimental observations that are incompatible with classical expected utility theory. In 2017, Richard Thaler was awarded the Nobel Prize in Economics for his contributions to behavioral economics. His empirical and theoretical research is instrumental in creating a rapidly growing field.

The decision making process does not only include the optimization task, but a set of tasks, namely: the selection of the (stochastic) model; the collection of data; the parametric, semi-parametric or non-parametric estimation of the model; the choice of the appropriate optimization criterion and finally the determination of the numerical solution of the optimization problem. The accuracy of the last task depends on the quality of the chain of previous subtasks. Since Ellsberg's paradox (1961), economics has been enriched by several models of decision making in an uncertain environment. Some models have integrated ambiguity into decision making and have proposed to integrate the individual's attitude towards ambiguity into the utility function. According to Ellsberg (1961), the term "ambiguity" often refers to the ambiguity of probabilities in an economic setting, and typically to uncertainty in the values of financial parameters at the practice level.

Apart from the ambiguity, which can considerably affect the individual's choice, the latter can experience feelings about the decision itself (fear, anxiety, etc.) and feelings that he or she may experience afterwards ("relief" following a good outcome or "sadness" following abad outcome). Indeed, the decision-maker can compare the performance of the chosen alternative with that of other non-chosen options. This comparison can generate unfavorable feelings, called "regret" (Bell, 1982; Loomes and Sugden, 1982). Regret is the emotion that has received the most attention from theorists since most people can easily recall situations in which a poor decision led to painful regret. Bell (1982) and Loomes and Sugden (1982) defined regret as a consequence of making a decision in a risky setting and indicated that this feeling can occur when individuals appear, after the decision has been made, to have made the wrong decision, even though the decision appeared to be the right one at the time it was made.Bell (1982) and Fishburn (1982) established the theory of regret, which was later developedby Loomes and Sudgen (1982, 1986) and axiomatized by Quiggin (1994). Since the early 1990s, several psychologists and economists have studied the role of the feeling of regret in the decision-making process (Simonson, 1992; Larrick, 1993; Boles and Larrick, 1995; Zeelenberg, 1999; Connolly and Zeelenberg, 2002; Inman and Zeelenberg, 2002; Buther and Connolly, 2006; Pieters and Zeelenberg, 2007). Thus, a large literature has shown that the feeling of regret has an impact on the individual's investment choice (Gilovich and Medvec, 1995; Zeelenberg et al., 1998, 2000; Camille et al., 2004; Coricelli et al., 2005).

Our work is part of the general framework of behavioral finance and particularly of portfolio optimization based on decision criteria that go beyond the standard expected utility. Indeed, this paper aims in particular at addressing the following problem:

What is the impact of ambiguity aversion on the optimal portfolio profile?

- To answer these questions, our article is organized in two parts:

The first axis deals with two essential points and deals with the transition from modern theory to behavioral portfolio management theory. It focuses mainly on the theoretical aspect where we give a general overview of the theories of decision making under uncertainty and the criticisms of the classical theory.

The second axis focuses on portfolio optimization in the context of static portfolio management in classical and behavioral finance.

First, we present the model of modern portfolio management. We apply our sample data, which is the estimated and annualized returns of three indices (S&P500, Euro Stoxx 50 and SSE Composite) over a period of 10 years (from December 1, 2008 until January 31, 2019), the study does not deal with the periodof COVID 19 and this to allow consistent results and not impacted by the health crisis. Secondly, we study the optimization of standard portfolios in the framework of behavioral finance.

1. the genesis of behavioral portfolio management theory

The theory of consumer choice under risk and uncertainty is part of the field of microeconomics. Risk and uncertainty are sometimes substitutable terms, but their meaning isnot always easily understood. Knight (1921) proposed in his treatise "Risk, Uncertainty and Profit" a fundamental distinction between risk (the probability of occurrence of a known event) and uncertainty (the probability of occurrence of an unknown event). Decision-making under risk and uncertainty has been and continues to be a very important subject of study, and has been the subject of much empirical work in order to develop decision models that are consistent with observed behavior. Numerous models of decision making that rely on the probabilistic rationality of the individual exist in the literature. Fermat and Pascal were the first to introduce the notions of probability and mathematical expectation in the mid-17th century. Their discoveries made it possible to rationalize certain choices. However, they did not make it possible to account for certain behaviors. This field was subsequently explored by Bernoulli (1738), who introduced the concept of expected utility to better represent certain behaviors. This theory, which is based on the calculation of probabilities and which corresponds to a uniform attitude towards risk (concave utility function), was developed by anaxiomatization of choices by von Neumann and Morgenstern in 1944.

Expected utility theory was then challenged by several researchers (namely: Friedman and Savage, 1948; Allais, 1953; Ellsberg, 1961; Markowitz, 1952) who proved that the axioms of the expected utility theory (independence axiom) are not consistent with real human behavior. Indeed, Friedman and Savage (1948) disagreed with this uniform attitude towards risk and pointed out that the majority of investors buy insurance and a lottery simultaneously. They proposed a utility function with a concave part (purchase of insurance) and a convex part (purchase of lottery). Thus, Markowitz (1952) showed that when individuals are faced with gains, they behave in the opposite way to those faced with losses. He pointed out that the curve falls very quickly in the loss part and increases relatively less quickly in the gain part. In addition, the expected utility theory has been criticized by several researchers in psychology, in terms of judgments and decision making.

1.1. Expected utility theory

Bernoulli (1738) introduced the notion of expected utility in order to solve the St. Petersburg paradox (1713), which assumes that the investor is willing to invest very large sums of money to participate in a game of chance. This seems to be unrealistic for Bernoulli (1738), who proposed to transform the monetary gains into a utility function that better represents the individual's satisfaction. This utility function is an increasing and concave function. In his work, Bernoulli (1738) used the logarithmic function of the type :

$$U(x) = \alpha \log(x).$$

Generally speaking, according to Bernoulli (1738), the evaluation of a lottery by the investor takes the following form:

$$\mathbb{E}(U(X)) = \sum_{i=1}^{n} p(x_i)U(x_i).$$

Where **X** is a lottery, **p(xi)** is the probability of occurrence of the outcome **xi ∈ X and U : X**

→ R is the utility function. This representation allows us to evaluate the lottery **X** using theexpected utility of the outcomes generated by the lottery.

1.1.1. Presentation of the expected utility theory

In 1944, von Neumann and Morgenstern (VNM) developed, in their book "Theory of Games and Economic Behaviour", the theory of expected utility, which aims at representing the investor's preferences in the form of a functional with values in the reals (this is called a cardinal representation of preferences). The latter must satisfy certain well-defined properties related to the axiomatics of preferences. This theory makes it possible to model the behaviorof the individual in risky and uncertain situations.

Consider a lottery X ∈ L with n outcomes defined by the vector X = (x1, p1; . . . ; xn, pn); where L is the set of possible lotteries; xi denotes the payoff for event i; pi is the probability of occurrence of event i such that ∀1 ≤ i ≤ n, pi ≥ 0 and i∑ pi = 1. The formalization of the rational investor's behavior is based on a set of axioms that deal with the binary preference relation :

- **Comparability** (also called completeness) **axiom**: The individual can always make a comparison between the available lotteries: let $X1 \in L$ and $X2 \in L$; the investor can prefer X1to X2 ($X1 \geq X2$); prefer X2 to X1 ($X2 \geq X1$) or be indifferent between X1 and X2 ($X1 \sim X2$);

- **Axiom of transitivity (or consistency)**: Let $X1 \in L$;$X2 \in L$ and $X3 \in L$ be three lotteries. Then if $X1 \geq X2$ and $X2 \geq X3$, we necessarily have $X1 \geq X3$;

- **Continuity axiom**: Let $X1 \in L$, $X2 \in L$ and $X3 \in L$ be three lotteries such that $X1 \geq X2$ and $X2 \geq X3$. Then there exists a real number $\alpha \in [0, 1]$ such that : $X2 \sim \alpha X1 + (1 - \alpha)X3$. This axiom leads to the existence of a utility function $U : L \to R$ that verifies: $X1 \geq X2 \Longleftrightarrow U(X1) \geq U(X2)$. This axiom allows us to infer that small changes in the probabilities of eventsoccurring do not change the individual's preference order.

-Independence axiom: Let $X1 \in L$;$X2 \in L$ and $X3 \in L$ be three lotteries and $\alpha \in [0, 1]$, such that $X1 \geq X2$. Then we always have $\alpha X1+(1-\alpha)X3 \geq \alpha X2+(1-\alpha)X3$. This shows that introducing a new lottery to an existing set of lotteries does not influence the individual's preferences.

1.1.2. Criticisms of the standard expected utility theory

Standard expected utility theory has been widely used by many researchers in the context of decision making. However, this theory has been challenged several times. Indeed, many authors such as Allais (1953) and Ellsberg (1961) have shown that the behavior of agents does not conform to the axioms of this theory, especially the independence axiom.

The Allais paradox (1953): In his 1953 paper, Allais experimentally demonstrated situations in which individuals do not make choices consistent with what they should do if their preferences verified the basic axioms of the von Neumann and Morgenstern theorem (1944). Indeed, Allais (1953) showed the violation of the independence axiom of the expected utility theory. Allais (1953) showed that the actual behavior of individuals is not consistent with expected utility theory. This is a violation of the independence axiom. This is due to the principle of regret which is not consistent with the principle of expected utility theory (Loomes and Sudgen, 1982).

Ellsberg's paradox (1961): Ellsberg contradicts rational choice theorists by showing that individuals do not react in the same way to objective and subjective probabilities. Indeed, some people tend to avoid ambiguous situations and prefer unambiguous ones. Furthermore,in 1961, Ellsberg defined ambiguity aversion behavior. Based on his experimental experiments, Ellsberg (1961) showed that individuals reject ambiguous situations. In other words, he found that individuals prefer to bet in a scenario where the probabilities of occurrence of the different possible alternatives are known. Thus, several researchers have questioned the classical expected utility theory and have presented several alternative approaches related either to the distortion of probabilities or to the distortion of expected utility.

Friedman and Savage's paradox (1948): Friedman and Savage (1948) developed a utility function modeling the behavior of individuals under uncertainty. From their experimental studies, they found that there is a dichotomy between individuals who buy insurance and individuals who buy lotteries. Indeed, most individuals buy insurance and lotteries simultaneously. This shows that most investors are both averse and risk-takers. Friedman and Savage (1948) proposed a utility curve that describes the attitudes of individuals towards risk in different socio-economic groups. This curve describes the behavior of investors who buy insurance and participate in a game of chance simultaneously. Curve 11 is composed of three parts: two concave curves and one convex curve, so that the convex curve links the two concave curves. This utility curve is plotted in (wealth, utility) space as follows:

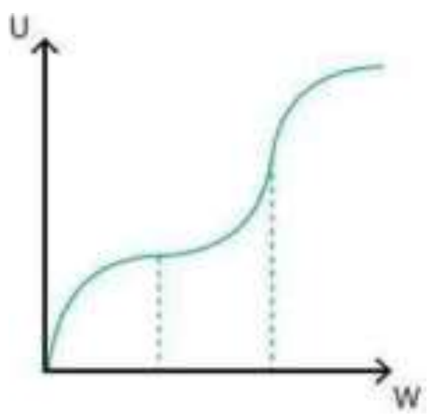

FIGURE 1: Friedman and Savage (1948) utility function

Each part of the curve corresponds to a given socio-economic class: the first part of thecurve is reserved for low-wealth investors; the second part of the curve is reserved for individuals with intermediate wealth; and the third part represents high wealth individuals. However, Friedman and Savage (1948) found many differences between individuals, even within the same socioeconomic group. Some are risk takers, while others are risk avoiders.

1.2.1. Alternative approaches to expected utility theory: Non-expected utility

Until the 1970s, economists focused on the rationality of homo-economicus. Indeed, expected utility theory was the most important theory used in the context of choice in the presence of risk and uncertainty.

However, since its creation, this theory has always been subject to theoretical and empirical developments that have led to the violation of axioms, notably the independence axiom (the Allais and Ellsberg paradoxes) and to the appearance of new alternative approaches. Although the expected utility function helps individuals to understand the real world, it is important to remember that this function is only a simplification of it. Indeed, several economists and psychologists have shown that individuals' behaviors do not necessarily conform to the expected utility theory and that the expected utility theory does notfully reflect the way individuals interact in the real world. Thus, several experimental proofs, including that of Kahneman and Tversky (1979), have violated the conventional axioms of expected utility.

These violations indicate that several factors that are likely to influence individuals' choices are neglected or poorly specified by conventional theory. In this context, new approaches have been developed to improve the decision-making process, based in particular on the deformation of objective probabilities.

1.2.1 Prospect Theory (Kahneman and Tversky, 1979) Prospect theory is an alternative theory to the expected utility theory of von Neumann and Morgenstern (1944). It is one of the first to have integrated the irrational behavior of the individual in an empirical way. This theory is one of the most important for describing the decision-making process in an uncertain future. Prospect theory is a founding theory of behavioral finance. Within the framework of prospect theory, Kahneman and Tversky (1979) determined the individual's satisfaction by the variation of the final return with respect to a reference point and not by a final wealth.

Figure 2: Decision-making process according to prospect theory.

Kahneman and Tversky (1979) assume that the choice process is divided into two phases: an editing phase and an evaluation phase. The editing phase makes it possible to organize and reformulate the different alternatives in order to simplify the evaluation of the subsequent phase. This phase consists of applying operations that transform the probabilities of the returns associated with the different perspectives. The first operation in this phase is coding: the investor defines the outcome as a gain or loss based on a reference point. The second operation is combination: this step consists of simplifying the prospects by combining the probabilities associated with the same outcome. The third operation is separation: this consists of separating the non-risky components of the outlook from the risky components. In the second phase, the edited prospects will be evaluated and the prospect with the highest value will be selected. Indeed, Kahneman and Tversky (1979) have developed a specific model for the evaluation and selection of edited perspectives.

1.2.2. The cumulative perspective theory (CPT)

The development of prospect theory has sparked the interest of many practitioners and researchers to apply this theory in many areas. However, this theory has been criticized because of two problems. It is not applicable for perspectives with a large number of realizations. In addition, it does not always satisfy stochastic dominance. To solve these problems, Kahneman and Tversky developed in 1992 a new version of prospect theory that applies to any number of risky or uncertain prospects: it is the cumulative prospect theory (CPT). Kahneman and Tversky (1992) conducted empirical studies focusing on the investor's attitude towards risk. They found that: (1) the individual evaluates a lottery in terms of changes in wealth and not in terms of the absolute level of wealth attained; (2) losing an amount has a greater psychological effect on the individuals' well-being than winning an equivalent amount; (3) the individual tends to overweight events with a low probability of occurrence and underweight events with a medium probability of occurrence; and (4) the individual is a risk taker in the loss part and is risk averse in the gain part.5 These results enabled Kahneman and Tversky (1992) to introduce the CPT. This theory is based in particular on the transformation of the probabilities of the outcomes according to Quiggin's (1982) rank-dependent expected utility model (RDEU) and the use of different cumulative functions for gains and losses.

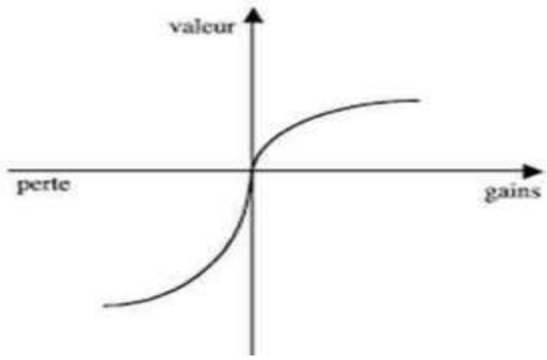

Figure 3: CPT Evaluation Function

The CPT function is concave in the gain part ($x \geq x*$) and convex in the loss part ($x < x*$). However, the slope of the curve in the loss part is steeper than that of the concave curve. This theory has been widely used in behavioral portfolio management.

1.3. The theory of ambiguity

1.3.1. Definition of the concept of ambiguity

The concept of ambiguity refers to a lack of precision or clarity of a statement or information in a specific situation. It refers to the diversity of possible interpretations of a word, an expression or a situation. This concept was introduced by Ellsberg (1961) in an article criticizing Savage's (1954) theory of subjective expected utility. In the context of economic decision-making, there are several situations where the information available to us to make a decision is vague, imprecise or ambiguous. Indeed, decision-makers are often confronted with a complex economic environment where it is difficult to assess the probabilities of outcomes. They may have doubts about their own assumptions and the rules that govern their world and guide their decisions. These individuals may know the nature of their models, but may not have enough information to determine the relevance and accuracy of each model.

Since Ellsberg (1961), the term "ambiguity" often refers to the ambiguity of probabilities in an economic framework. In practice, ambiguity typically refers to uncertainty in the values of financial parameters. The subject of imprecision in the information needed to make a decision was discussed by Ellsberg (1961) who highlighted the concept of ambiguity based on the following experiment: an individual faces two urns: a "risky" urn containing 50 red balls and 50 black balls, and an "ambiguous" urn also containing 100 balls of these two colors but with unknown proportions of red balls and black balls. It is assumed that this individual wins if he draws a ball of a specific color and that he can choose the urn in whichhe draws. In the "risky" urn, the probability of drawing a black ball is known: it is 0.5. In the other urn, the probability of drawing a black ball is unknown: it belongs to the interval [0, 1]. The results of this experiment show that people would rather bet on an urn in which the proportions of red and black balls are known than on an urn in which the proportions of red and black balls are unknown. This is because when people are faced with a choice between two options, the majority choose the one whose probability distribution is known.

1.3.2. Definition of ambiguity aversion

Ellsberg (1961) showed that individuals exhibit behavior that is commonly interpreted as "ambiguity aversion. In other words, his experiments showed that agents prefer to bet on unambiguous events, i.e., they choose the option that has the least amount of unknowns. Indeed, an ambiguity-averse individual tends to avoid uncertainty and adjust his or her behavior in favor of risks with known probabilities, even at significant costs. Indeed, individuals are always willing to invest significant amounts of money to avoid ambiguous processes in favor of normatively equivalent risk processes (Becker and Brownson, 1964; Keren and Gerritsen, 1999; Chow and Sarin, 2001). In the economic setting, ambiguity aversion has been employed to explain phenomena such as: the risk premium puzzle (Maenhout, 2004; Collard et al., 2011; Gollier, 2011; Ju and Miao, 2012) and the stock market participation puzzle (Dow and Werland, 1992; Easly and O'Hana, 2009). In addition, Alary et al. (2013) and Snow (2011) have shown that ambiguity aversion influences the choice of optimal insurance. These different theoretical contributions assume a universally negative attitude toward ambiguity.

Ambiguity aversion can be modeled from a second-order probability distribution. This means that the event in question is not characterized by a single probability but by a set of values. Moreover, it is well known that the probability of occurrence of an ambiguous event isa random variable and that its probability is therefore also random whose values are included in the interval [0, 1] (it follows a probability law). In this framework, a random probability is assigned to each of the possible values of a specific event. This is why it is called a second- order probability distribution. The strength of ambiguity aversion depends on the level of knowledge of the probability of payoff and whether the ambiguous alternatives are presented alone or side by side. For example, one piece of experimental psychology was able to show that ambiguity aversion was significantly stronger when there was an information conflict rather than an information gap and when the conflict emanated from different sources (Smithson, 1989).

1.3.3. The fundamental models of ambiguity theory

Ellsberg's (1961) mental experiment has given rise to several models incorporating theindividual's attitude towards ambiguity. Among the most developed models, we can cite the models introduced by Schmeidler (1989) and Gilboa and Schmeidler (1989). These works established the formal basis of the notion of ambiguity by modifying the theory of subjective expected utility of Savage

(1954).

-Approach of Gilboa and Schmeidler (1989): In parallel to the generalization of the expected utility model, in an uncertain context, Schmeidler (1989) proposed a generalization of the "subjective expected utility" model of Savage (1954). This generalization is called "Choquet Expected Utility" (CEU). The idea of this theory is to model the behaviors observedin the Ellsberg (1961) experiment. The axioms of this approach require that preferences be established according to the "roulette" type lottery9. This presents a considerable limitation. However, Gilboa (1987) presented an axiom without this limitation. He presented a model in which objective probabilities are absent and acts are established according to "horse race" type lotteries. The proof of Savage (1954) can be used to establish additive probabilities for unambiguous events. Schmeildler (1989) considers that sub jective beliefs about the probabilities of events are represented by non-additive probabilities. Faced with an ambiguoussituation, Gilboa and Schmeidler (1989) showed that the individual does not have a single belief (a unique subjective distribution), but rather a set of beliefs (a second-order subjective distribution). They introduced the "multiple priors approach".

This approach assumes that in the presence of ambiguity, the decision maker cannot determine a single probability for a state of nature. Indeed, he is faced with a set of possible probabilities on which he must make his choice. In this framework, Gilboa and Schmeidler (1989) developed the "maxmin expected utility" approach. In this approach, the decision maker has a set of probability laws and uses the "maxmin expected utility" criterion to evaluate decisions with respect to an initial set of beliefs.

-Approach of Maccheroni et al (2006). Maccheroni et al. (2006) developed anapproach that allows to determine the preferences of an individual, under ambiguity, by introducing the utility function U and the ambiguity index C defined on the set of probabilities of random events.

Let X and Y be two random variables representing possible choice outcomes. These outcomeshave values in this interval [-M, M], such that :

$$X \succcurlyeq Y,$$
$$\Leftrightarrow Min_{\mathbb{P}\in\Delta} \int U(X)d\mathbb{P} + C(\mathbb{P}) \geq Min_{\mathbb{P}\in\Delta} \int U(Y)d\mathbb{P} + C(\mathbb{P}),$$

where Δ denotes a convex set, U corresponds to the modeling of the individual's attitude toward risk, C represents the individual's attitude toward ambiguity. This presentation encompasses the "maxmin expected utility" model of Gilboa and Shmeidler (1989) and the multiple preference model of Hansen and Sargent (2001).

2. Standard portfolio management in the context of behavioral finance

Traditional economics presents results and recommendations whose values depend on the ability of individuals to collect and process information in an optimal way. This discipline is based on the rationality of actors. Since the 1960s, social psychology, cognitive psychology and experimental studies have demonstrated the importance of cognitive biases1 and heuristics2 in the decision-making process. This has necessitated the introduction of a new form of economics that takes into account the role of emotions in decision making, called "Behavioral Economics". The Nobel Prize in Economics awarded to Daniel Kahneman in 2002 for his pioneering work with Vermon Smith. This is the founding work of prospect theory, which is the basis of behavioral finance. Kahneman (2002) showed that individuals are not rational even if they are reasonable in their choices and decisions. Indeed, decisions are made on the basis of errors of judgment and calculation that reveal deficient heuristics.

Behavioral finance, which is a component of behavioral economics, is a new approach that has emerged to address the difficulties encountered by the traditional paradigm in financial markets. Indeed, behavioral approaches study the way individuals behave based on experimental evidence. These approaches allow us to explain real-life events better than traditional finance. Behavioral finance explores the behavior of individuals in a direct or indirect way by examining their different minds, desires, mistakes, preferences, and behaviorsthrough experimental tests. Sewell (2010) showed that behavioral finance is the study that focuses on the influence of psychology on the behavior of financial agents and its subsequent effect on the market. Thus, behavioral finance is considered the study that focuses on how psychology influences the behavior of individuals, at the individual or group level, in financial markets.

2.1. Presentation of the sample

2.1.1. Description of the sample

In this empirical part, our study focuses on the monthly prices of 3 stock market indexes: S&P 500; Eurostoxx 50 and SSE Composite over a 10-year period, from December1, 2008 to January 31, 2019, not including the COVID 19 period, in order to allow for more meaningful results not impacted by the health crisis. This allows us to have 121 observations for each of these indices. First, the S&P 500 (Standard & Poor's 500 index), founded in 1957, is an index of the market capitalization of the 500 largest publicly traded U.S. companies in terms of market value. It is composed and weighted by the S&P Dow Jones Index. Due to its diversity, the S&P 500 Index is one of the most common benchmarks for the U.S. equity markets. It is considered an indicator of the US economy. Second, the Euro Stoxx 50 Index, founded in 1998, is a market capitalization-weighted index of the 50 largest European companies operating in the Eurozone. It includes 50 of the most representative companies in the European markets, which are composed of 18 different economic sectors. The componentsof the Euro Stoxx 50 Index are selected based on a number of criteria. For example, they are weighted according to free float market capitalization. The Euro Stoxx 50 Index is considered the benchmark index for the European markets. Finally, the SSE Composite Index (Shanghai Stock Exchange Composite Index), founded in 1990, is a stock market index that is composedof all A and B class stocks listed on the Shanghai Stock Exchange in China. It is considered the representative index of the Shanghai Stock Exchange. This index is a good way to get an overview of the performance of companies listed on the Shanghai Stock Exchange.

2.1.2. Estimation and annualization of data

Using the prices of the 3 indices (121 prices for each index), we calculated the monthly returns of each index using this formula:

$$R_i = \ln(\frac{S_{it}}{S_{it(t-1)}}); \; i = 1, 2, 3;$$

Where Sit is the price of index i at time t. This calculation allows us to have 120 returns for each stock.

The prices of three stock indices (S&P 500; Euro Stoxx 50 and SSE Composite) follow the geometric Brownian motion, such that:

$$\frac{dS_i}{S_i} = \mu_i dt + \sigma_i dw_t^i \; pour \; i = 1, 2, 3;$$

Where μi is the trend of index i and σi is its standard deviation. In this framework, the returns of these indices at maturity T are given by :

$$R_T^i = \exp[(\mu_i - \frac{1}{2}\sigma_i^2)T + \sigma_i w_T^i].$$

Let us note by : $\alpha_m(i) = \mu_m(i) - \frac{1}{2}\sigma_m^2(i)$, with : and $\mu_m(i)\sigma_i^2$ are respectively the monthly trend of index i and the monthly variance of index i.

The standard deviation of the index i is given by :

$$\sigma_{auuel}(i) = \sqrt{12}\sigma_m(i),$$

And the trend of the index i is given by :

$$\mu_{auuel}(i) = 12 \times \alpha_m(i) + \frac{1}{2}\sigma_{annuel}^2(i).$$

Finally, the expected return of the indices over a one-year horizon is given by :

$$E(R_p) = \exp(\mu_{annuel}(i)),$$

The variance of the indices over a one-year horizon is given by :

$$Var(R_p) = \exp(2\mu_{annuel}(i)(\exp(\sigma_{auuel}^2(i) - 1))).$$

2.2. The modern "mean-variance" portfolio approach

At this level, we first present the "mean-variance" model of Markowitz (1952). Then, we determine the set of possible portfolios. Then, we establish the Markowitz (1952) efficientfrontier and the efficient portfolio, based on the historical data of our sample.

2.2.1. Presentation of the "mean-variance" approach.

Generally speaking, the investor's problem is to determine the optimal asset allocation.Markowitz (1952) showed that the investor seeks to optimize his portfolio choice by taking into account not only the expected return on his investment, but also the risk of his portfolio (the variance). To this end, he developed the modern portfolio theory, called the "mean- variance" approach. This theory is based on how the investor can construct an efficient portfolio that either maximizes the expected return on the investment for a given level of risk, or minimizes the risk for a given level of expected return. This can be illustrated through thesetwo equivalent optimization programs:

$$\max_w \mathbb{E}[R_P],$$

S.c $Var(R_P) = \sigma^2.$

And in an equivalent way:

$$\min_w Var(R_P),$$
$$\text{S.c. } \mathbb{E}[R_P] = \overline{R},$$

Where is R_P the portfolio return, σ^2 is the level of fixed risk and is $\overline{R}$ the level of fixed expected return.

The resolution of the above optimization program allows us to have the set of efficient portfolios. The set of these portfolios allows us to establish a hyperbola, called the "efficient frontier". This frontier is represented by portfolios whose composition allows to optimize the couple (the expected return and the risk). These portfolios allow for a maximum expected return for a given level of risk or a minimum level of risk for a given level of expected return.

2.2.2. The determination of the set of possible portfolios and the efficient frontier.

To determine the set of possible portfolios, we solve the following optimization problem:

$$\max_w w'\overline{R} - \frac{\phi}{2} w'\Sigma w,$$
$$\text{S.c. } w'e = 1 \text{ et } w \geqslant 0,$$

Where $\overline{R} = E(R_P)$ is the vector of annual expected rates of return of the indices and $\Sigma = Var(R_P)$ is the annual variance-covariance matrix of the indices' returns. Using the historical monthly data of our sample, the vector of monthly expected returns and the monthlyvariance-covariance matrix of the 3 indices are given by respectively:

$$\overline{R}_m = \begin{pmatrix} 0,0012 \\ -0,002 \\ -0,001 \end{pmatrix},$$

$$\Sigma_m = \begin{pmatrix} 0,0042 & 0,001 & 0,0002 \\ 0,001 & 0,0039 & 0,0018 \\ 0,0002 & 0,0018 & 0,0067 \end{pmatrix}$$

We notice that the expected rates of return of both stocks (stock 2: Eurostoxx 50 and stock 3: SSE Composite) are negative. This result allows us to break out of the classical framework. Indeed, this result can be explained by the fact that a good part of the data in our sample covers the period of the 2008 crisis. This seems to be more relevant and allows us to have a more realistic investor behavior.

Our study focuses on the annual values of the vector of expected returns and the variance- covariance matrix. In order to measure the profitability of each of the securities in our sample as a function of its level of risk, we used the profitability measure "Sharpe ratio" expressed by

: $S_i = \dfrac{R_i - r}{\sigma_i}$ avec $i = 1, 2, 3;$

Where Si is the Sharpe ratio of security i; Ri is the return of security i; r is the risk-freerate and σi is the volatility of security i. For a risk free rate r = 2%, the results of the Sharpe ratio for the 3 securities are given through the following table:

	Titre 1 (S&P 500)	Titre 2 (Euro Stoxx 50)	Titre 3 (SSE Composite)
Ratio de Sharpe	−0,2918	−0,3541	−0,2563

Table 1: Sharpe ratio of the three securities

The results in Table 1 show that all three stocks in our sample have negative Sharpe ratios. We notice that stock 1 has the largest Sharpe ratio, followed by stock 3 and then stock

2. This shows that stock 1 in our sample outperforms the other two. In order to determine the optimal portfolio weights, we determine the investor's risk aversion φ which yields the optimal portfolio weights vector. For this, we rely on the study by Palma et al. (2009)3 on the estimation of the risk aversion coefficient of women and men. Their results are illustrated in the following graph:

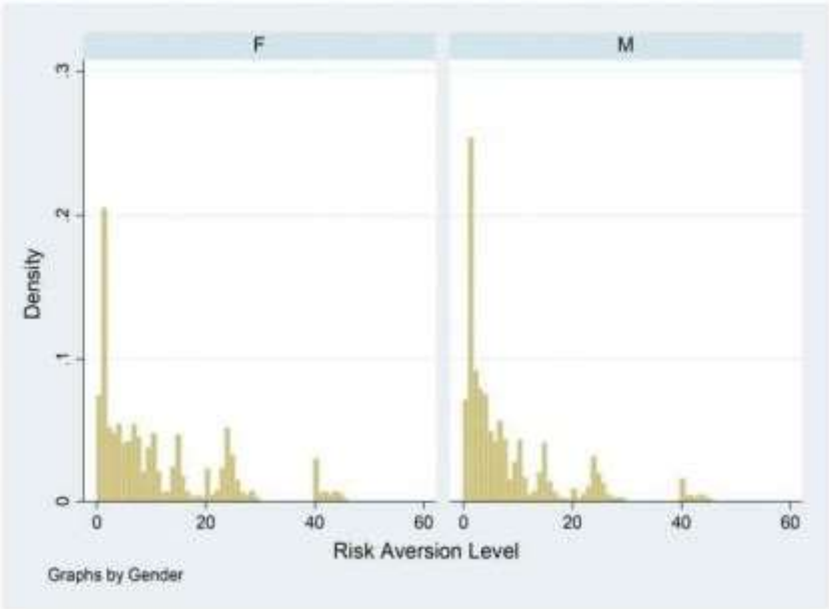

Figure 4: Individual's level of risk aversion

The graph shows4 that the majority of the sample studied (women and men) have a risk aversion between 1 and 10. Indeed, Palma et al (2009) found that less than 21% (25, 5%) of women (men) have a risk aversion below 2 and almost 10% (6%) have a risk aversion above 25. These results allowed us to take different values of the investor's risk aversion coefficient φ between 1 and 10, such that φ takes the following values 1,2,5 and 10. We apply the model of possible portfolios on our sample by considering several values of the risk aversion coefficient (φ = 1, 2, 5, 10). Table 2 below shows the weights of the 3 indices in our sample for the different values of risk aversion:

ϕ	Rendement espéré	Risque	Titre 1 (S&P 500)	Titre 2 (Eurostoxx 50)	Titre 3 (SSE Composite)
$\phi = 1$	1,0149	0,0043	0,9999	0	0
$\phi = 2$	1,0149	0,0043	0,9999	0	0
$\phi = 5$	1,0149	0,0043	0,9998	0	0,0002
$\phi = 10$	1,0111	0,0034	0,8567	0,0001	0,1432

Table 2: Security weights in the efficient portfolio.

The results in Table 2 show that when the risk aversion coefficient φ is equal to 1,2 and 5 allows us to have degenerate results. Indeed, taking into account the constraint of positivity at the level of weights (w 0) allows us to avoid short selling situations. This explains the degenerate results. However, when the risk aversion coefficient φ is equal to 10, we obtain a non-degenerate efficient portfolio. These results allow us to set the risk aversion coefficient at φ = 10 for the rest of this section.

From Table 2, we notice that for low values of φ (φ = 1, 2), the investor invests all of his wealth in stock 1 (S&P 500), which is the only asset in our sample with a positive expected rate of return and represents the relatively low level of risk. For a slightly higher level of risk aversion (φ = 5), we notice that the investor invests almost all of his wealth on stock 1 (0, 9998) and a very small part (0, 0002) on stock 3 (SSE Composite), which represents the highest level of risk in our sample and negative expected rate of return but is more attractive than stock 2 (Euro Stoxx 50). Thus, we find that the levels of expected return and risk of the efficient portfolios remain constant for these three cases (φ = 1, 2, 5).

For a higher level of risk aversion, (φ = 10), we note that the investor allocates his wealth to the three stocks as follows: a large portion of wealth (85, 67%) is invested in stock 1(S&P 500), which has the highest expected return among the stocks in our sample and relatively low risk compared to the other stocks; 0.01% of the individual's wealth is investedin stock 2 (Euro Stoxx 50), which is the least risky stock but has the lowest expected return; and finally 14.32% of the wealth is invested in stock 3 (SSE Composite), which is the riskiest asset but has a negative expected return but is a little more attractive than the second stock Wesee that for a high level of risk aversion, the risk-averse investor no longer takes the risk of investing in a single asset and spreads his wealth over all available securities. All these resultsregarding the weights of the securities within the efficient portfolios seem logical since security 1 has the highest Sharpe ratio compared to the other two securities. We also note that the higher the risk aversion coefficient, the lower the levels of expected return and risk in the efficient portfolio. Thus, we find that the higher the risk aversion coefficient, the lower the proportion invested in the first stock (S&P 500) and the higher the proportion invested in the third stock (the riskiest stock).

Now, we assume that the investor's risk aversion coefficient is φ = 10, solving the Markowitz (1952) optimization program provided us with the set of possible portfolios which is 5146 portfolios. This result can be illustrated, in the standard deviation and expected return plane.

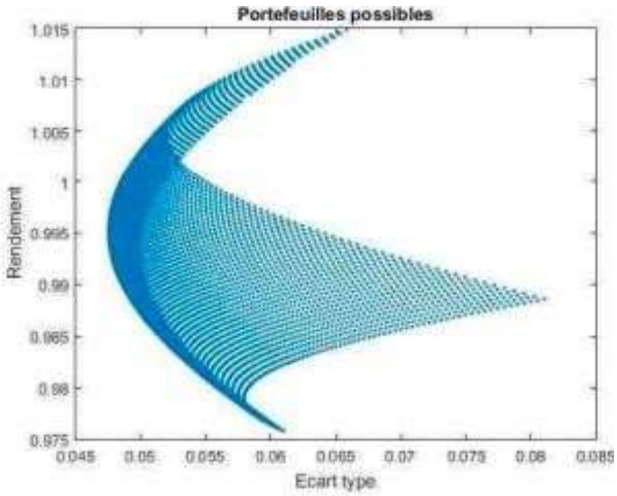

Figure 5: Set of possible portfolios

These results allow us to determine the matrix that corresponds to the weights of the 3 stocks in the 5146 possible portfolios. By combining the 3 indices, in order to build an efficient portfolio, the mean-variance approach generates several portfolios by varying the share of each security in the portfolio. This allows us to have the optimal portfolio combination that corresponds to a maximum level of return for a given level of risk.

2.3. Standard portfolio management in the context of behavioral finance

Several studies have shown limitations for modern portfolio management namely: (1) Lack of diversification: investors own a small number of assets (Berber and Odean, 2000); (2)Naive diversification: investors tend to invest their wealth uniformly across all assets (Benartzi and Thaler, 2001); (3) Local bias: the investor prefers to invest in local assets rather than foreign assets (French and Poterba, 1991); (4) Availability: investors tend to predict the probability of a return based on the amount of return on each asset that comes readily to mind (Kahneman and Tversky, 1973); (5) Emotional and cognitive: the errors that arise from the way people think (Bower and Wright , 1992) and (6) Gathering: describing how investors behave in a group without any planning (see work by Schiller, Nobel Prize in Economics in 2013). Modern portfolio theory cannot explain these anomalies because it assumes that the investor is rational and has complete information and that investor preferences are very well defined. However, behavioral portfolio theory is a violation of "mean-variance" portfolio theory and can explain investor behavior. Shefrin (2002) defined behavioral portfolio theoryas an application of psychology to behavioral finance. In this framework, we determine the different standard portfolio management work for an individual investor according to the different behavioral approaches.

2.3.1. Behavioral portfolio theory (BPT):

In 2000, Shefrin and Statman proposed the behavioral portfolio theory (BPT). This theory takes into account the feelings of investors when choosing the optimal portfolio. BPT is based on Roy's (1952) Safety-First approach and incorporates certain behavioral, economic and financial characteristics. This theory combines two theories of choice in the presence of uncertainty: the SP\A theory (Lopes, 1987) and the cumulative prospect theory (Kahneman and Tversky, 1992). In their model, Shefrin and Statman (2000) assume that the investor seeks to maximizethe expected return on the portfolio by keeping the probability of bankruptcy below a given level α. Their model is given by:

$$\max \mathbb{E}(R),$$
$$\text{s.c. } P(W < s) < \alpha,$$

Where R is the portfolio return; s is the subsistence level; α is the allowable bankruptcy leveland W is the final wealth.

Shefrin and Statman (2000) presented the BPT in two versions: the first version is Single Mental Account BPT (BPT-SA), where the investor integrates his or her portfolio intoa single mental account, and the second version is Multiple Mental Account BPT (BPT-MA) where the investor integrates his or her portfolio into multiple mental accounts.

The BPT-SA approach. First, Shefrin and Statman (2000) developed the version of BPT-SA in which the covariances between financial assets are not zero. This approach is based on the SP/A theory. The investor considers his portfolio as a single mental account portfolio. In this framework, the selection of the optimal portfolio under BPT-SA is similar to that of the mean-variance approach in several respects. This is because the investor prefers theportfolio with the highest expected return and the lowest risk $P(W \leq A)$. Therefore, the BPT- SA efficient frontier is obtained by maximizing the expected portfolio return $Eh(W)$ for a fixed value of $P(W \leq A)$.

The BPT-MA approach. In a second step, Shefrin and Statman (2000) developed the BPT-MA case, where the investor integrates his portfolio in several mental accounts and where the covariances between these different accounts are neglected. Indeed, the BPT-MA isbased on mental accounting and prospect theory (Kahneman and Tversky, 1979) allowing investors to make decisions based on gains and losses determined relative to a reference point.

The BPT-MA approach assumes that the investor views his portfolio as a multi- layered pyramid where each layer is associated with a particular objective. Thus, the investor's attitude towards risk changes from one layer to another. In general, most investors combine the desire to have a low level of aspiration with the desire to have a high level of aspiration. Shefrin and Statman (2000) found that the portfolio that combines low and high aspiration takes the form of a pyramid, with the first layer at the base being "downside protection" and designated to avoid poverty and the layer at the top being "upside potential" and designated to enable the investor to achieve wealth. This corresponds to the behavior of an investor who simultaneously buys insurance and a lottery.

2.3.2. Standard portfolio optimization under ambiguity theory.

In economic analyses, several choice situations are characterized by uncertainty or ambiguity. These situations are different from situations characterized by risk. Ellsberg's paradox (1961) showed that this difference is significant at the behavioral level. This suggeststhat there are two dimensions of the decision maker's beliefs about the probabilities of events occurring: risk and ambiguity. The concept of ambiguity refers to a lack of precision or clarityof a statement or information in a specific situation. It refers to the diversity of possible interpretations of a word, an expression or a situation. In standard models, ambiguity is neglected by the individual.

Merton (1980) found in his study that it is difficult to anticipate the expected return on stocks and that the individual needs a very long time to estimate this return. In order to incorporate ambiguity in decision-making situations, Chen and Epstein (2002) proposed a multiple-priority utility model. Hansen and Sargent (2001) and Anderson et al. (2003) are the first to propose studies that allow for ambiguity in portfolio optimization. Liu (2008) used an exogenous set of priors and a utility of multiple-priors developed by Chen and Epstein (2002) to capture the notion of ambiguity and ambiguity aversion. Later, Fei (2007) determined the investor's optimal portfolio by considering the investor's expectation and ambiguity. He foundthat ordinary martingale methods and Malliavin's computational methods can be used to solvethe investor's optimization problem. He also showed that anticipation and ambiguity affect theinvestor's choice of optimal portfolio. Pflug and Wozabal (2007) determined the optimal portfolios when the underlying probability model is not perfectly known. They applied the "maxmin" approach that explicitly allows for ambiguity in the choice of the probability model. This model is called "the robust optimization model". It allows the use of a confidenceset for the probability distribution. Pflug and Wozabal (2007) have shown that the monetary value of information in their model can be determined.

Estimating the parameters of a model is always very difficult. Moreover, the modern optimization model is sensitive to the estimates of the input parameters. Consequently, the performance and composition of optimal portfolios obtained using the mean-variance approach are always sensitive to perturbations in the main parameters of the model (e.g., the expected return and the variance-covariance matrix). In alternative theories to the classical expected utility theory, many authors assume that investors have the same opinion about the true probability distribution of random events. However, uncertainty in the values of the financial parameters leads to misspecification. To address this problem, Hansen and Sargent (2001) proposed a robust control model. The robust preference approach considers that the individual's objective function takes into account the possibility that the model used by the individual may be wrong and is only an approximation of the real model. Hansen and Sargent (2001) showed that uncertainty can be based on ambiguity that results from a lack of information about randomness.

In the context of asset allocation, the investor faces not only the risk of the return on financial assets but also ambiguity in the vector of expected returns and/or in the volatility andcovariance of asset returns. The question of uncertainty in the portfolio optimization model has been the subject of much research. In this context, the robust optimization method has been proposed. This approach is considered as a new modeling tool. It proposes vehicles to integrate the risk related to the estimation of input parameters in the decision-making process in the framework of asset allocation.

Conclusion

Behavioral finance challenges the classical financial theory based on investor rationality, from which the notion of an efficient market is derived. It takes into account potential market inefficiencies and the hypothesis of investor irrationality. In a nutshell, behavioral finance consists in better integrating psychology into finance. This new science makes it possible to explain financial market anomalies caused by human behavior. These causes are related to psychological factors that interfere with decision making, namely ambiguity aversion, regret aversion and disappointment aversion. These biases have led to the emergence of several alternative utility theories to the classical utility theory. These theories have been applied in the context of standard and structured portfolio management. Within thisframework, several alternative models to the models of modern finance, which allow for the consideration of investors' anticipated feelings, have emerged. Within the framework of this behavioral vision of portfolio choice theory, we have tried to answer the problem of our research, namely the impact of ambiguity aversion, regret aversion and disappointment aversion biases on the optimization of standard and structured portfolios, including in the multidimensional case. In this way, we determined the positioning of the optimal portfolio in the presence of ambiguity and examined the impact of risk aversion and ambiguity aversion on the profile of the optimal portfolio.

Portfolio optimization in the standard framework originated with the mean-variance approach (Markowitz, 1952). This approach has been the basis and starting point of several portfolio optimization models. However, this model has been criticized by several behavioral finance researchers as not meeting the requirements of real individuals. In this work, we have studied portfolio optimization in the static framework according to the mean-variance approach and in the behavioral finance framework by taking into account different behavioral biases. First, we presented the mean-variance model and applied our sample data to the resultsof this model in order to establish the weights of the efficient portfolio for different risk aversion levels (1, 2, 5, 10). We have shown that for a relatively high level of risk aversion equal to 10, the result is not degenerate. For a risk aversion level equal to 10, the results showed that the efficient portfolio is well diversified. This portfolio is essentially composed of the stock with the highest expected return and the relatively lowest risk level among the stocks in our sample. We also established the efficient frontier and the set of possible portfolios corresponding to this level of risk aversion.

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